is increased until the next constructive interference is achieved, the change in frequency, Δf, is given by

$$\frac{4\pi L}{v} \Delta f + \Delta\phi_r = 2\pi \qquad (2)$$

where $\Delta\phi_r$ is the accompanying change in ϕ_r. Thus, if $\Delta\phi_r$ is known or can be neglected, the velocity, v, can be determined by measuring Δf; Δf can be determined by directly measuring the frequencies at successive peaks and troughs [*Spetzler*, 1970], but their positions may be affected by the amplitude envelope and by noise. Instead, the phase of the sinusoid in Figure 3(a) was calculated continuously as a function of carrier frequency [*O'Connell et al.*, in preparation]. The phase of this sinusoid is directly related to the phase difference between superposed echoes in the sample; thus pressure derivatives can also be determined by measuring the phase as a function of pressure.

The first step in calculating the phase is to filter out the amplitude envelope and any harmonics and noise from the data. This is done via the spectrum (Figure 3b) of the data calculated by the Fast Fourier Transform (FFT) algorithm, and the result is illustrated in Figure 3(c). The filtered data are phase shifted by 90°, via a Hilbert transform in the frequency domain, and the "instantaneous" phase is then given by the inverse tangent of the ratio of the transformed to the original filtered data. The process is described in more detail by *O'Connell et al.* [in preparation]. The calculated variation of phase with carrier frequency is illustrated in Figure 3(d), and residuals from a best-fit straight line are shown in Figure 3(e). If reflection phase shifts in the sample could be neglected, the slope of this line would be a measure of the sound velocity in the sample. Small systematic deviations from linearity can be seen in Figure 3(e), which are interpreted below as arising from reflection phase shifts in the sample.

IV. TRANSDUCER-BOND PHASE SHIFTS

A sound wave reflected from a sample face to which a transducer is bonded is actually the superposition of waves reflected from three faces: the sample-bond and bond-transducer interfaces and the outer face of the transducer. The phase and amplitude of the resultant wave depend in a complicated way on the thickness and relative acoustic properties of the bond, transducer, and sample. This dependence, for the case of plane, parallel waves and interfaces, has been given by *Redwood and Lamb* [1956], who used the analogous theory of transmission lines [see also *McSkimin*, 1957; *Williams and Lamb*, 1958]; identical results can be obtained by considering plane elastic waves directly. The total transducer phase shift is

$$\phi_{tf} = 2\psi_{tf} - \pi \tag{3}$$

where

$$\tan \psi_{tf} = \frac{Z_f}{Z_s} \frac{Z_t \tan \theta_t + Z_f \tan \theta_f}{Z_f - Z_t \tan \theta_t \tan \theta_f} \tag{4a}$$

$$= \frac{Z_f}{Z_s} \tan (\eta_t + \theta_f) \tag{4b}$$

and

$$\tan \eta_t = \frac{Z_t}{Z_f} \tan \theta_t \tag{5}$$

Here, Z_s, Z_t, and Z_f are the acoustic impedance of the sample transducer, and bond ("film"), respectively, and θ is the thickness, in terms of the phase of the sound wave, of the transducer or bond

$$\theta = kl = 2\pi l/\lambda = 2\pi lf/v \tag{6}$$

where k is the wave number, λ is the wavelength, and l is the thickness. Subscript t or f in (6) would indicate transducer or bond properties, respectively. The acoustic impedance, Z, of a medium is the product of the density, ρ, and sound velocity, v, of the medium

$$Z = \rho v \tag{7}$$

The total reflection phase shift, θ_r, between successive echoes in the sample is the sum of (3) and a phase shift of π occurring on reflection at the other end of the sample. An example of calculated phase shifts as a function of carrier frequency is illustrated in Figure 4 for the case of a 10 Mhz quartz P-transducer (X-cut) bonded to a (100) face of MgF_2. The relevant acoustical properties are listed in Table 2, along with those of other materials discussed in this paper. Different bond thicknesses are represented by the parameter

$$\tau_f = \frac{l_f}{v_f} \tag{8}$$

which is the transit time (in nsec) of the wave through the bond. Wave velocities in typical bond materials are 1 to 2 km/sec. For v_f = 1 km/sec, τ_f is the bond thickness in μ.

For zero bond thickness, the phase shift is symmetrical about multiples of the transducer resonance frequency (in this case, the third), and equation (4) simplifies to